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Cross-cultural differences in representations and routines for exact number

Michael C. Frank

Stanford University

The relationship between language and thought has been a focus of persistent interest and controversy in cognitive science. Although debates about this issue have occurred in many domains, number is an ideal case study of this relationship because the details (and even the existence) of exact numeral systems vary widely across languages and cultures. In this article I describe how cross-linguistic and cross-cultural diversity—in Amazonia, Melanesia, and around the world—gives us insight into how systems for representing exact quantities affect speakers' numerical cognition. This body of evidence supports the perspective that numerals provide representations for storing and manipulating quantity information. In addition, the differing structure of quantity representations across cultures can lead to the invention of widely varied routines for numerical tasks like enumeration and arithmetic.

1. Introduction.[1] The relationship between language and thought is one of the most fascinating—and the most controversial—topics in cognitive science. Posed by Whorf (1956), the question of whether cross-linguistic differences lead to differences in cognition has been studied extensively across a wide range of domains. Recent work on this question has come from color perception (Kay, Berlin, Maffi & Merrifield 2003, Winawer et al. 2007, Roberson & Henley 2007), navigation and spatial language (Hermer & Spelke 1994, Levinson, Kita, Haun & Rasch 2002), theory of mind (Pyers & Senghas 2009), gender

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(Boroditsky, Schmidt & Phillips 2003), event perception (Papafragou, Hulbert & Trueswell 2008, Fausey & Boroditsky 2011), object individuation (Lucy 1992, Barner, Li & Snedeker 2010), categorization (Lupyan, Rakison & McClelland 2007), and many others. Yet despite considerable empirical progress, the general form of the relationship between language and thought remains hotly contested (Davidoff, Davies & Roberson 1999, Gentner & Goldin-Meadow 2003, Gumperz & Levinson 1996, Levinson et al. 2002, Li & Gleitman 2002, Pinker 1994).

Numerical cognition—and specifically, the use of language to represent large, exact quantities—is an exciting case study of this relationship in a domain that is both cognitively central and at the core of many technical achievements. Although there has been considerable discussion of the role of grammatical number marking as a case study of language and thought (e.g. Barner et al. 2010), the ability to represent arbitrarily large, exact numbers may have somewhat larger cultural and technical consequences. Hence, this review will cover only conventionalized representations that are suitable for representing large quantities—numbers like "seven" or "thirty-four"—and the routines that allow us to use them.[2]

The goal of the review is to give a sketch of some cross-cultural evidence on the relationship between numerical representations and routines. Rather than attempting to perform a comprehensive review of ethnographic evidence, I will instead focus primarily on recent psychological work that uses experimental methods in the field. Although there is tremendous value in linguistic and ethnographic work on number—and I discuss some in the final sections—my hope is to highlight how cross-cultural experiments can sharpen hypotheses about the relationship between language and thought by providing measurements of behavior in situations where numerical representations vary.

The outline of the review is as follows. I begin by describing background on representations and routines for number. I then present studies on numerical cognition in the absence of linguistic representations of numbers (evidence from Amazonian languages) and cases where language for number is culturally available but either not available to individual speakers (in Nicaraguan signers and home-signers) or not available online (in the moment in which a task is being performed). This body of evidence supports the idea that storing and manipulating exact quantity information depends on having both a representation of quantity and a routine for the appropriate task available in the moment when they are needed. I finish by surveying some examples of how number representations can vary due to cultural demands (examples from Melanesia) and how routines can vary depending on the structure of the representations they operate over (focusing on mental abacus users in India).

Taken together, the evidence supports a view that my collaborators and I have referred to as the "cognitive technology" view (Frank, Everett, Fedorenko & Gibson 2008, Frank,

[2] The term "number" is generally ambiguous between grammatical markings like singular/plural and numerals that describe the exact cardinality of sets. Here I will avoid the cumbersome language necessary to disambiguate in every instance and use the terms "numbers" and "numerical cognition" under the assumption that these terms refer to numerals representing the exact cardinalities of large sets and the broad range of cognitive operations that are carried out with such sets, respectively.

Fedorenko, Lai, Saxe & Gibson 2012): that numerical representations are cultural artifacts that are used for the online encoding of quantity information. The form of a linguistic or cultural representation of number and the efficiency of the routines for manipulating this representation each affect what computations are possible using this representation; the online availability of this representation (in the moment a computation is desired) is a prerequisite for performing the computation. One version of this view was first articulated by Kay and Kempton (1984) and it and its variants are currently experiencing a resurgence in cognitive science (Dessalegn & Landau 2008, Gentner 2003, Wiese 2007); see e.g. Frank et al. (2012) for more detailed discussion.

A secondary goal of this review is to argue for an approach whereby fieldworkers supplement standard elicitation techniques with psychological experimentation that tests the cognitive consequences of different numerical representations and routines. Because of the immense linguistic and cultural diversity in regions like Amazonia and Melanesia and the relative isolation of these populations, investigation of numerical systems in these regions' indigenous cultures provides especially rich evidence regarding the range of variation in number systems. Melanesia, in particular, is likely to harbour the greatest diversity of number systems in the world (Lean 1992). Ethnographic observation and psychological observation can play complementary roles in characterizing this diversity, providing both naturalistic observations and precise and generalizable measurements. And given the rapid decreases in linguistic diversity in these regions (Evans 2009a), it is especially important to document not only the facts of languages in Amazonia and Melanesia, but also the psychological consequences of these languages for their speakers.

2. Representations and routines for number. The past twenty years have seen an explosion of interest in representations of exact number as an example of an important, uniquely human concept, yet one that is built out of primitive components that can each be observed in infants and members of other species (Dehaene 1997, Carey 2009). On the one hand, numbers are a key part of every modern society: they facilitate a huge set of human behaviors, from complex feats of engineering to economic exchanges using currency. On the other, representations of quantity information can be observed in infants, monkeys, fish, and a host of other creatures (Gallistel 1993, Xu & Spelke 2000, Hauser et al. 2003). Thus, in the domain of number, cognitive scientists can ask how basic cognitive abilities can be combined into a sophisticated conceptual system and, in particular, what role language plays in this combination.

The basic cognitive systems that provide non-verbal representations of quantity are now well established (Feigenson, Dehaene & Spelke 2004). The first is a system that can track the location and identity of up to three or four objects at a time, likely based in visual attention or tracking. The second is the approximate number system (ANS), which can represent the approximate magnitude of sets of objects but not the identities of individuals within these sets. Despite the presence of both of these systems in prelinguistic infants, learning how to use linguistic numerals is a protracted process. In typically-developing English-speaking children, the time period from learning the meaning of "one" to mastering the use of number words up to "ten" can last a year or more (Wynn 1990).

Despite consensus about the basic facts, the role of language is contested in both this developmental progression and its end result. On the "bootstrapping" account, learning the meanings of numerals in the count list is a result of first mapping number words from "one" up to "three" or "four" onto small number representations, and then performing an inductive step that recognizes the parallel between the sequential relationship between the words in the count list and the sequential relationship inherent in their definitions. The specifics of language—both in the structure of the count list and in the use of number names as placeholders for concepts—play an essential role in this account (Carey 2009, Piantadosi, Tenenbaum & Goodman 2012). In contrast, the "mapping" view suggests that words like "four" or "seven" are defined in terms of innate number concepts, and identified either noisily, using the ANS, or precisely, using a count routine. On this kind of account, language plays a peripheral role: it does not help to create new concepts, it simply helps to name and recognize pre-existing concepts by using enumeration routines like counting (Gelman & Gallistel 1978).

One broad area of agreement between these views, however, is the distinction between numerical representations and numerical routines, and the importance of their interaction in allowing their users to store and manipulate exact quantities (Gelman & Butterworth 2005, Carey 2009). By numerical representation, I mean here a set of symbols used for the task of representing exact quantities. The choice of a representation of number includes the medium of representation (linguistic, like a count list; externalized, like a counting stick; or even supported by visual imagery, like a mental abacus representation) and the internal structure of these representations (e.g. that English speakers say "ninety-nine" = 90 + 9 to mean 99, while French speakers say "quatre-vingt-dix-neuf" = 4 * 20 + 10 + 9). By numerical routine, I mean an algorithm that is commonly used to leverage such a representation in a particular numerical task. Examples of routines range from simple enumeration to the complex sets of steps that schoolchildren are taught to follow in order to perform addition or division of large quantities.

3. Numerical abilities without representations of exact number. What is numerical cognition like in the absence of linguistic numerals in a language?[3] Are there any routines for manipulation of exact quantity that are possible in the absence of exact numerical representations? This section reviews recent work with the Mundurukú and Pirahã, two indigenous groups in Brazil, that explores the cognitive consequences of speaking a language with limited or no vocabulary for exact quantities.

3.1. Measuring number vocabulary. Gordon (2004) claimed that Pirahã had a counting system consisting of words for the quantities 1 (*hói*) and 2 (*hoí*) as well as a word for "many" (*aibaagi*).[4] He reported data from only a single elicitation (in which a speaker

[3] The question of what it means to have exact numerals in a language is ambiguous: an individual speaker can in principle have access to a particular, idiosyncratic mapping between symbols and quantities; or a mapping can be conventionalized and available to many or all speakers of a language. Although there are cases of idiosyncratic or heterogeneous number systems (for preliminary data on this issue, see e.g. Frank & Honeyman 2011), the examples discussed here all show relatively broad consensus across speakers, shown via experimental procedures used with a sample of individuals from the community.

[4] Here and throughout the article I will use the Arabic numerals as a shorthand for the expression "the quantity N" regardless of whether the quantity is large or small, rather than following standard typographical conventions ("one" vs. 11) depending on quantity. I will quote numbers like "seven" to refer to a word for a quantity.

used the "two" word *hoí* to refer to the quantities 3 and 4). These data were broadly in accordance with a description of Pirahã as a "one, two, many" language, a type found in other non-industrialized societies (Menninger 1969, Hammarström 2010).

In their work on Mundurukú, Pica, Lemer, Izard, and Dehaene (2004) performed a structured elicitation experiment. They presented sets of 1–15 dots in random order to adults and children and asked how many dots were present in each set. Mundurukú participants responded consistently with a set of conventionalized terms for the quantities 1–3. These terms were used by participants in nearly all cases. For 4, participants used a conventional term almost as often, but occasionally used the same term to refer to 5 and 6. For 5, 25% of participants used a term meaning "one hand," while 35% of others used a vaguer term that Pica and colleagues translated as "some, not many" and that was used for other quantities 5–15 as well. Above 5, only this latter term and a term meaning "many" were used with any frequency. This experiment gives evidence that Mundurukú does have some exact numerals, but lacks a recursive number naming system and exact number vocabulary for large quantities.[5]

Following on Gordon (2004), our own work revealed a different view of Pirahã quantity vocabulary, using a structured elicitation task like Pica et al. (2004). We showed participants sets of objects and asked "how much/many are there?", increasing the cardinalities of the set from 1–10 and then decreasing from 10–1 (or vice versa). We found that the quantities for which our participants used particular words changed depending on the context of the elicitation (increasing vs. decreasing). In particular, although participants used *hói* only for 1 in the increasing context, they used it for up to 6 objects in the decreasing elicitation. This context effect strongly suggests that *hói* is not a word for 1. On our view, the most likely conclusion from these data is that it is a relative term like "few," "fewer," or even "small." Another possible position, however, is that *hói* is polysemous between "one" and "a few"; this view is of course logically possible, but provides no account of why or under what conditions an exact meaning would be available. The three words documented by Gordon are confirmed by several non-native Pirahã speakers to be the only words for quantities, leading us to conclude that Pirahã seems to have no (unambiguous) words for exact numbers: not even a word for 1.

The Amazonian findings suggest that representations of exact quantities are not a linguistic universal. In addition, they raise the intriguing question of whether any other languages without numerals have been misclassified as "one, two, many" languages due to the absence of experimental data.[6] In order to determine the semantics of possible numerals, single-participant elicitations should be replaced with structured elicitations and numeral comprehension tasks (Wynn 1990). Even data for a handful of participants in

[5] Note that for developmental researchers, the gold standard for children having acquired the meaning of a numeral for 7 is success in comprehension-based tasks like "give a number" (Wynn 1990, Le Corre et al. 2006, Condry & Spelke 2008). In the "give a number" task, participants are simply asked to "give me N objects" and the cardinality of the set they give is reported. Neither the Mundurukú nor the Pirahã have been tested on such a task, so more work remains to be done to probe the meanings of the attested vocabulary items.

[6] Hammarström (2010) gives a list of other languages that have such systems and notes this possibility, though Pirahã may be the only one of these that lacks any singular-plural marking as well.

such tasks can be informative and can provide an inexpensive supplement to current field methods.

3.2 Consequences of limited number vocabulary. In contrast with linguistic representations of number, which vary across societies, a large body of evidence shows that an approximate number sense (ANS) is available to all human beings as well as members of other species. This approximate sense leads us to be able to make estimates of a set's quantities without using an enumeration routine.

The ANS has been characterized extensively in human and non-human animals (for review see Feigenson et al. 2004, Gallistel 1993). Estimates of quantity made by the ANS follow Weber's law (e.g. Whalen, Gallistel & Gelman 1999, Xu 2002), which states that the probability of a correct response in a discrimination task is related to the magnitude of the stimulus being discriminated. Weber's law leads to the prediction of the relation $\sigma/\mu = c$ in participants' data, where μ and σ are the mean and standard deviation of the magnitude estimates (across trials or participants) and c is a constant holding across a range of magnitudes. The term c is often referred to as the *coefficient of variation* or COV. A constant COV implies that the larger the quantity being estimated, the larger the average error, in turn signaling that the ANS is being used.

In Pica et al.'s study, Mundurukú participants and French controls performed comparison, addition, and subtraction tasks. When participants were asked to choose the larger of two large sets of dots (and were not given enough time to count), both groups performed similarly, showing a constant COV, consistent with Weber's law. However when participants were asked to give the resulting quantity in a subtraction paradigm where objects were first added to and then subtracted from an opaque container, French participants performed nearly perfectly, while the Mundurukú made errors that were again consistent with the operation of the ANS. Crucially, the design of this task required only responses in the range where the Mundurukú could have responded verbally (quantities 0–2), ruling out the explanation that they could not indicate the correct response even though they knew it.

Like the Mundurukú, the Pirahã also relied on the ANS to perform numerical tasks. Gordon (2004) performed a range of matching tasks designed to probe the ability of participants to store and manipulate exact quantities. In the simplest task, participants were asked to produce a 1–1 match between two sets by selecting the correct quantity of objects to align with a target set. In more difficult tasks, the target set was presented in a cluster or was presented only briefly, and participants were again asked to produce a target set of the same cardinality. Participants made errors in all tasks, even the 1–1 match task, although their errors were larger in those tasks where the target set was presented for a short period of time. When Gordon consolidated data across all tasks, the pattern of responses again showed a constant COV. Like the Mundurukú results, these findings suggest that analog estimation using the ANS is the default strategy in situations where no count list is available.

Both sets of results left open an important question, however: did Mundurukú and Pirahã participants understand that large quantities *could be* exact, even if they did not know how to express or manipulate them? For example, Gordon's 1–1 matching task was the simplest task in either assessment, yet Pirahã still made errors. Were these errors due to confusion about what was being asked or difficulties in completing the task, or were they instead due to a more fundamental conceptual difference? On the first interpretation, the Pirahã made errors in matching up larger quantities of objects either because they did not understand that an exact response was called for (even though they could have produced such a response) or because they made manual errors in alignment even though they understood what was being asked of them. On the second interpretation, however, the Pirahã did not understand that a correct response required matching exactly, because they did not even have available a concept of exact equivalence.

The actual computational demands for success in the 1–1 matching task are quite low. In order to succeed, it is only necessary to match individuals until there are no more left to match. This task can be accomplished without ever representing the total quantity, so success in the task does not demonstrate the existence of exact quantity representations. A 1–1 match of exactly 7 items can be performed without ever mentally representing 7. On the other hand, a true failure in the task—an inability to select the 1–1 matching algorithm, even with appropriate training and unlimited time—would suggest that the Pirahã truly did not think in terms of exact equivalence or exact matches.

On a recent visit to the Pirahã, my collaborators and I replicated a number of Gordon's tasks with a larger sample of participants (N=14, as opposed to N=5 in the previous study). In order to ensure task understanding, we included a systematic training phase in which we demonstrated what the correct response would be for one trial with a small quantity and then gave corrective feedback on another set of small-quantity trials until participants were performing consistently (Frank et al. 2008). In the more difficult matching tasks, we found precisely the pattern of ANS usage that Gordon documented, with errors increasing along with the quantity of objects being estimated (see figure 1 for an example of the testing environment). Our results differed from Gordon's in the 1–1 matching task, however. There, only one participant made any errors and the rest performed perfectly, suggesting that this task was qualitatively different from the others. Despite not having linguistic representations of exact quantities available to them, this group of Pirahã understood that an exact response was required. This result shows that our participants made the appropriate generalization from a few training examples with small numbers: that every target item should be matched with *exactly* one item, not that the two sets should match approximately. That they made this generalization consistently across individuals strongly suggests that the notion of an exact, rather than approximate, 1–1 match was available to them (though again, not the representation of a particular exact quantity like 7).

One final dataset bears on this question, however. Everett and Madora (2012) conducted a replication of our previous work with another group of Pirahã from a different village. Although they again replicated the pattern of ANS usage on more complex matching tasks, they found results congruent with Gordon's: their participants made systematic errors on the 1–1 matching tasks. Everett and Madora argued that the success of the particular participants in our 2008 experiments was due to exposure that members of this village had to innovated number words and numerical procedures. Apparently, Madora had conducted numerical training sessions with the members of this village; nevertheless, our elicitation tasks showed no evidence for knowledge of innovated number words. This claim brings up an interesting possibility: could it be that exposure to some representations of exact number— even without the long-term adoption of these representations—facilitates the construction of a 1–1 match strategy? Although the current data do not provide enough information to evaluate this claim, perhaps it can be assessed via future developmental or cross-cultural work.

FIGURE 1. A Pirahã participant in Frank et al. (2008), in the orthogonal match condition. The experimenters have placed 10 spools of thread, and the participant has matched them with 9 balloons.

To summarize, evidence from the Pirahã and Mundurukú demonstrates that in cultures without representations of large exact quantities, individuals are not able to remember or manipulate such quantities exactly, suggesting a connection between linguistic representations and the ability to create routines for manipulating exact number. Instead of remembering exact quantities, both groups used an estimation strategy which allowed for approximately correct responses even in relatively difficult tasks. Nevertheless, evidence from the Pirahã suggests that it is possible to create and use a routine for exact, 1–1 match even without an unambiguous linguistic representation of 1.

4. DISTINGUISHING COGNITION FROM CULTURAL EXPOSURE IN NUMBER REPRESENTATION. The evidence above suggests that routines for storing and manipulating exact quantities correlate with the cultural presence of linguistic representations of number, but the precise nature of this correlation is unknown. One possibility is that language for number could simply co-occur with cultural routines for number, rather than being a causal factor in the cognition of individual speakers. On this kind of account, language for number would develop alongside a set of (possibly non-verbal) routines for manipulating exact quantities, springing from the same basic cultural needs. Speakers would learn number words, but they would also learn algorithms for doing matching tasks, for chunking large quantities into sets of smaller quantities, and for tallying to keep track of quantities over time. For example, the use of an abacus would constitute a parallel, non-linguistic routine that could support numerical calculation (see below for more details). On the other hand, another possibility is that language for number could be necessary in the moment for the precise manipulation of exact quantities: that is, language could be a necessary constituent in these routines (like in the case of verbal arithmetic, but unlike in the case of an abacus).

Recent studies have begun to differentiate between these two accounts. First, work with signers in Nicaragua has investigated the numerical abilities of individuals in a highly numerate culture who nonetheless have limited representations of exact number and limited routines for manipulating these representations. Second, psychophysical experimentation with verbal interference tasks has begun to manipulate the online availability of linguistic representations of exact number in highly numerate, educated adults. These two sets of studies are reviewed below.

4.1. Cultural exposure alone does not scaffold exact number. Nicaraguan Sign Language (NSL) is a new sign language created over the last 30 years as specialized schools have brought together the community of deaf individuals in Nicaragua (Senghas, Kita & Ozyurek 2004). As the Nicaraguan deaf community has grown and the age at which children are exposed has become younger, NSL has evolved into a fully-featured, highly grammaticized language that includes number words, complex spatial language (Senghas & Coppola 2001) and sophisticated constructions for reporting the thoughts of others (Pyers & Senghas 2009).

Since NSL speakers live in a numerate community, playing gambling games and using money, they have ample opportunities to acquire numerical routines. Nevertheless, number signs in NSL underwent rapid standardization in the early 1990s, transforming from iconic finger signs—with a number of fingers corresponding to the quantity being indicated—to a set of simpler, one-handed signs that are less iconic. This change has created a population of speakers with a range of experience with numbers signs: there are older adults who did not learn either system as children; younger adults who learned the iconic system but have since learned the second system; and adolescents who learned the second system during childhood (Flaherty & Senghas 2011). By keeping cultural exposure relatively constant but varying linguistic representation, the case of NSL thus presents an opportunity to test whether cultural exposure to numerical routines is sufficient for accurate performance of numerical tasks or whether it is necessary to have linguistic representations in order to acquire or carry out numerical routines.

Flaherty and Senghas (2011) tested NSL speakers across the full range of ages on a set of tasks that included matching tasks like those used by Gordon (2004) as well as tasks requiring tapping out quantities, counting and selecting sets using number words, and translating between monetary notes and coins. Across all tasks, the group that made far and away the most errors were the older adults that had not fully mastered even the iconic count lists. Individuals who had mastered either count list made small but systematic errors—indicating that they were not perfectly accurate in using their count routine in challenging situations— but the performance of older adults who could not count differed significantly from even that of the older adults who had been able to master the iconic count routine.

In addition, as with the Pirahã, all NSL participants—even the non-counters—succeeded in grasping the simplest 1–1 matching tasks. When matching tasks became more complex and the stimuli being matched were presented ephemerally (via tapping, or via putting items one by one into an opaque cup), accuracy was considerably lower for the non-counters. The non-counters knew that there was something they did not know, however—they expressed uncertainty about larger quantities, and had developed heuristic strategies for making change in the monetary tasks. They knew that an exact answer was required, but did not know how to calculate that answer. Thus, like the Pirahã, NSL speakers without a count routine were able to select an exact quantity matching strategy, even in the absence of a reliable method for mentally representing individual quantities.

Although many deaf children in Nicaragua are now given opportunities to learn NSL, there are still some individuals who have not had access to the broader deaf community and have instead built up more idiosyncratic sign systems for communicating with their families and more immediate community. "Homesign" systems of this sort and their relationship to conventional language have been studied extensively, in the US and around the world (Goldin-Meadow & Mylander 1984). Recent work by Spaepen, Coppola, Spelke, Carey and Goldin-Meadow (2011) investigates numerical cognition in Nicaraguan homesigners. Congruent with the work with NSL speakers, Spaepen and colleagues found that homesigners, who could not produce a consistent count list or perform matching tasks, were still able to compare monetary denominations with high accuracy.

In addition, although they could not produce a correct ordering of number signs, the homesigners did still know words for exact quantities. This knowledge allowed Spaepen and colleagues to perform an important exact numerosity recognition task. In this task, the homesigners were told that some exact number of objects were in a box, and then the array in the box was transformed (either via a change in the number of objects or not). When the transformation did not change the quantity in the box, the homesigners almost always used the same gesture as the experimenter; when the transformation did change the quantity, they never used the same gesture. Ruling out a pragmatic explanation for this behavior (e.g., applying the principle of contrast; Clark, 1988), nearly all participants used gestures that matched the direction of the transformation, for example signaling a larger number than the original gesture when an object had been added to the set. This task gives clear evidence that the homesigners understood that each set had an exact numerical value, even if they did not have an errorless routine for finding that value.

Although both NSL users and homesigners grew up in a highly numerate culture, this fact alone did not create the concepts and routines necessary to succeed in complex exact number tasks. In addition, supporting the Pirahã 1–1 matching results, the Nicaraguan data suggest that neither number words nor a count routine are necessary to understand the idea that a set has an exact quantity, even if that quantity cannot be named or stored in memory.

While the Nicaraguan data implicate linguistic representations (rather than cultural exposure to routines) as playing a causal role in the ability to manipulate exact quantities, it is a separate question whether this role is *online*. In other words, for an individual with a lifetime of practice representing exact quantities, does representing a quantity like 7 require the use of language in the moment such that if linguistic resources were not available at that moment, this task would become much more difficult or impossible? To answer this question, we turn to psychophysical tasks performed with numerate English speakers.

4.2. Number words must be available online for enumeration. Verbal interference methods have been used widely for testing the online dependence of various tasks on language (Newton & de Villiers 2007, Winawer et al. 2007, Hermer-Vazquez, Spelke & Katsnelson 1999). Verbal interference refers to a class of experimental paradigms in which participants are asked to perform a task while simultaneously occupying their verbal system by performing a separate verbal task, such as repeating a word like "the," repeating strings of numbers, or "shadowing" (repeating words after immediately after hearing them spoken on a recording). As a control for the generalized dual-task cost of performing two tasks at once (Pashler 1994), performance in the target task under verbal interference is often compared to performance in the target task paired with a non-verbal task like shadowing a clapped pattern.

A handful of studies have used verbal interference to measure numerical behavior. However, most have done so using number tasks that were themselves verbal. For example, Logie and Baddeley (1987) found that rapid repetition of "the" caused more errors in counting than either listening to speech or tapping a finger, suggesting that active speech production interfered with use of the same system to count. A more recent study by Cordes, Gelman, Gallistel and Whalen (2001) showed an Arabic numeral and asked participants to press a key that number of times while either repeating "the" or counting very quickly. They found that participants under verbal suppression showed a constant coefficient of variation—indicating use of the ANS—while those who were counting showed a decreasing COV (perhaps caused by the binomial errors implied by skipping numbers in the count list). These two studies give evidence that language interference does cause participants to make errors when aspects of the task are linguistic, but leaves open the possibility of better performance in purely non-linguistic tasks.

In order to test this possibility, my colleagues and I conducted a series of experiments where we replicated the matching tasks used with the Pirahã, performing these tasks with a group of English speakers who were simultaneously shadowing complex texts (Frank et al. 2012). This paradigm had the benefit of using a purely nonverbal measure of number knowledge and of providing data that could be compared directly to those collected during fieldwork with the Pirahã. Our results suggested strong parallels between the performance of the English speakers—who did not have number language available in the moment—and that of the Pirahã—who had never known words for numbers. Like the Pirahã (and Nicaraguan populations), the English speakers under verbal interference were able to do the 1–1 matching task with relatively few errors. In addition, the English speakers, like the other populations, showed evidence of relying on the ANS in the hardest matching tasks. Followup experiments using matched verbal and spatial memory interference tasks showed that this pattern was specific to language interference.

However, the English speakers also showed some differences from the Pirahã. In the medium-difficulty matching tasks where there were visual cues (e.g., matching the quantities of two orthogonal lines), they made errors but their overall performance did not show the signature of the ANS (a constant relationship between the quantities being matched and the magnitudes of the errors in estimation). Instead, the magnitude of the errors increased with respect to the quantity being matched. We posited that their errors resulted from the use of ad hoc matching routines like making correspondences between sub-groups of objects. This same pattern of increasing errors was observed in the Nicaraguan signers who did know a count list, indicating that this pattern of data may generally result from the application of fallible routines.

More broadly, the picture that emerges from the evidence so far suggests an online, causal role for language in the representation of number information. Evidence gathered through psycholinguistic fieldwork, in combination with laboratory control tasks, suggests that representing 7 requires having some internal symbol like "seven" available in the moment. This pattern of evidence should not suggest that there is no role for cultural needs in the creation of numerical routines, however. The next section gives several ethnographic examples of interactions between culture and numerical representations and routines.

5. Numerical representations and routines can be shaped by culture. A common perspective on English numerals—even from sophisticated, numerate adults—is that they are transparent linguistic tools that do not reflect an idiosyncratic evolutionary process driven by specific cultural needs. Yet a closer look at the diversity of count systems in the world's languages falsifies this view. While the examples discussed below only provide an existence proof for cultural effects on representational systems, it is a goal for future research to understand both the prevalence of such effects and the mechanisms by which cultural demands can lead to representational innovations. For example, Wiese (2007) gives an account of how number concepts and numerals evolve in concert; her ideas leave open several places where specific cultural demands could lead to particular representational idiosyncrasies over the evolution of a count list. Thus, my hope is that discussing examples of possible links between culture and number representation can give some insight into how this relationship could function. I give three examples below.

First, the ways that numerals are named can change in response to the needs of individuals in a culture. For example, in Mangarevan, a language spoken on an island in French Polynesia, tools, breadfruit, and octopus are each counted with different sequences (Beller & Bender 2008). The Mangarevan language includes an abstract counting system that extends to high numbers, but it also includes three different systems for applying this list to different kinds of objects. These different systems rename the basic count unit to be groups of 2, 4, or 8 of an object, allowing for much more efficient grouping and counting of large numbers of objects. Beller and Bender argue that this division reflects a case in which a single number system has fragmented into a number of task-specific systems. Although each system incorporates properties of the more abstract count list, the need for greater efficiency and accuracy in specific situations led to the move away from a single, abstract system.

Second, the entire structure of a count system can be determined by a sufficiently important cultural practice. The vast majority of the world's count lists are structured around bases that are 5, 10, or 20 (Hammarström 2010), presumably because human beings have five digits on our hands and feet (and 20 digits overall). Base-5, base-10, and base-20 systems interact with and are supported by finger- and toe-counting routines. The languages of the Morehead-Maro region of Papua New Guinea have received considerable recent attention, however, because they are base-6, an extremely rare pattern (Donohue 2008, Hammarström 2009, Evans 2009b). Many of them include lexical items for relatively high exponents, e.g. up to 65 or 66 in Keraakie. Evans (2009b) and Hammarström (2009) give a compelling account of the origins of this system: it is specialized for the counting of yams, which can be arranged for storage in a petal-like configuration. In addition, in an interesting twist, the base-6 representation leads to a reinterpretation of finger-counting routines: these routines become base-6 as well, using the wrist as a final location and re-construing the finger count as a count of "attachment points" (finger joints and wrist joint) (Evans 2009b). In this way, finger-counting is reinterpreted with respect to the base-6 representation that has evolved (or been invented) to support an important cultural routine.

Third, changes to a numerical representation can also be motivated directly through changes in cultural routines. A specific example of this kind of comes from Saxe (1982). He documented that speakers of Oksapmin, a language spoken in the West Sepik province of Papua New Guinea, used a body-count system (a common type in the region (Lean 1992)). This count system was base-27, extending from one hand along the arm, over the head, and through the other arm to the other hand. However, when users of this count system had limited experience with manipulating money, they made systematic errors in simple addition problems (e.g. 8 + 6). Saxe found that although they could count out the first addend (8), these inexperienced users had not developed a correspondence strategy so that they could keep track of the number of body parts in the second addend (6).

Users more experienced with money manipulation had developed a number of strategies to circumvent this problem, however, including counting both the second addend and the sum of the addends in parallel, and splitting the body in two and using the second arm to track the second addend. The body-splitting strategy was most successful; it was used by the participants that were most experienced with money manipulation, but also required the most adaptation of the existing representation. To use it, Saxe's participants had to reverse the count list so that it could be initiated from either arm. Saxe's study beautifully demonstrates how cultural pressures can lead to the creation of new routines for arithmetic and can in turn lead to changes in the base representation.

Body count systems also suggest how the choice of a base—or more generally the design of a number representation—can interfere with the development of efficient routines. In the case of Oksapmin, the base was so high that the enumeration routine required both hands and hence could not be easily used to create two separate buffers for addition. This example is minor, however, compared with counting systems like one reported to be used by some speakers of One. This system, described by Donohue (2008), is in principle recursive and infinite, but in practice so cumbersome that it is rarely used to count quantities larger than a handful. One has individual lexical items for 1 and 2, but allows specific, conventionalized combinations of these words up to 6. Their count list admits the following combinations 1, 2, 2+1, 2+2, 2+2+1, and (2+1)+(2+1), but not (for example) 2+2+2. Although this system could be used to express 7, 10, or even 20, it quickly becomes impractical for larger quantities. This system may even be a recent innovation and hence indicative of a community whose use of numbers is in flux (Crowther 2001).

These examples give a flavor for the ways in which the vast range of number representations and routines in the world's languages can be shaped by their cultural context. Nevertheless, understanding the specific cognitive consequences of this variation will require significant experimental fieldwork, and the form of the relationship between particular numerical representations and the routines they support is mostly unknown. The last section of this review gives some evidence on this question by exploring a case study of a number representation that licenses a very different set of routines—in a different medium—from the others we have reviewed: mental abacus.

6. Non-linguistic representations: Effects of representation structure on routine. This review began by asking about the relationship between language and thought and explored this relationship through the diversity in number representations across the world's languages. But in fact there is a wide variety of non-linguistic representations used across different cultures. Aside from finger-count systems, Menninger (1969) describes the near-universal use in the ancient world of tally sticks and knot-based systems to keep track of large quantities. In many cases, tally-based systems evolved into the use of counting boards—devices that allowed for the grouping of tally objects like pebbles. Unlike most tally systems, counting boards incorporated the use of place value (in which a particular position in a notation stands for the order of magnitude of symbols in that position, e.g. $1= 10^1$, $10 = 10^2$, $100 = 10^3$), a major innovation that allowed them to be used flexibly for a wide variety of record keeping.

The modern Soroban abacus (primarily used in Japan and China) likely evolved from Roman counting boards. Like these counting boards, the Soroban abacus uses a base-10 representation with place values mapped to individual abacus columns. The abacus (and some of the most sophisticated counting boards) incorporates a subsidiary base-5 as well, however: on a standard Soroban each column has a single bead on top that represents 5 units in that place value, and four beads on the bottom that each represent 1 in that place value. Combinations of these beads allow the quantities 0–9 to be represented using a maximum of five beads.

Like tally sticks and other enumeration devices, counting boards and abacuses are external devices that allow their users to enumerate large exact quantities and retain them precisely over long periods of time. However, the abacus allows users to go beyond the simple enumeration task by allowing the development and use of efficient routines for arithmetic computation. Using the base-5 within-column representation and base-10 place value system, large computations can be broken into many small steps consisting of the addition of numbers below 5 and a corresponding set of "carry" operations (in which the parts of a result greater than 9 are transferred to the next highest place value). With practice, abacus calculations can become routinized and highly accurate. In a head-to-head competition in post-war Japan, a skilled abacus operator out-computed a calculator user (Kojima 1954). Crucially, abacus addition operates via a routine using set of memorized operations that are different from the commonly used base-10 addition operations.

Although abacus is an external computation aid, experienced abacus users can learn to internalize the abacus representation and make computations by manipulating beads on a mental image of an abacus. This technique, known as mental abacus (MA), is widely taught in Japan and has been the focus of recent interest in math supplementary education programs in Malaysia, India, China, and a number of other countries in Asia and the Middle East. Studies of MA have suggested that users do truly represent a mental abacus using visual imagery (Hatano 1977, Hatano & Osawa 1983, Stigler 1984). For example, they make off-by-5 errors far more than would be expected in standard linguistic calculation, indicating that they are inadvertently "dropping" the 5 bead from their mental representation. MA users also seem to be able to compute while performing linguistic distractor tasks (Hatano 1977) and neuroimaging studies confirm that MA activity induces activity in cortical areas related to visuo-spatial working memory (Tanaka et al. 2002, Chen et al. 2006). MA is also highly effective as an arithmetic method: a MA user took top honors in the 2010 World Cup of Mental Computation.

Our own recent work investigated how it is that MA representations are possible, given the attested limits on numerical representation in the visual system (Frank & Barner 2011). Neither of the two systems traditionally implicated in visual number processing (object processing for small numbers up to 4, approximate representations above that) would be able to represent a number like 49 on the abacus, since this would require representing the exact positions of 9 beads. Despite this, MA users are able to do impressive computations with far larger numbers. For a visual comparison of abacus computation and MA, see figure 2.

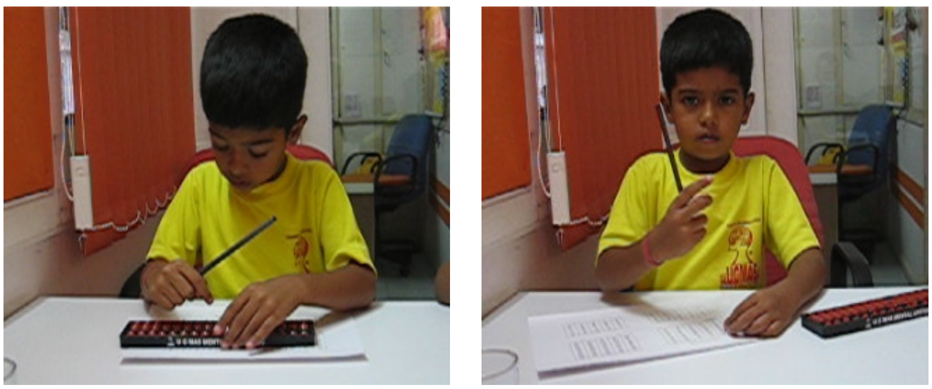

FIGURE 2. (left) A child performing a physical abacus computation. (right) The same child performing a mental abacus computation.

We tested a large group of children (ages 7–16 years) in Gujarat, India, who were enrolled in MA afterschool programs. We asked these children to do two standard MA tasks—addition of quantities and translation of abacus configurations into Arabic numerals—while we varied the difficulty of the tasks. In both tasks, we found that the limitations on performance came from the number of columns on the abacus representation—in other words, the maximum place value—rather than on other features of a given task. For example, many participants were able to add between 7 and 9 two-digit addends together in under 10 seconds, but very few were able to add 2 four-digit addends in the same time period. In contrast, the number of beads necessary to make a representation (e.g., whether a column showed a number like 0, with 0 beads, or 9, with five beads) in these problems did not seem to affect performance, once the number of columns was controlled.

To test the dependence of MA computations on language, we asked MA experts to perform verbal interference tasks as they did mental computations. While their performance was impaired slightly by verbal shadowing, they were if anything more impaired by simply tapping their fingers during the computation (presumably due to the reliance of the computation on the accompanying gestures, see figure 2). In contrast, a group of American college students—who used linguistic calculation strategies to do mental arithmetic—were highly impaired by verbal interference but experienced no interference from tapping their fingers.

The interference data suggest that MA is a fundamentally visual representation, while performance in the addition experiments described above suggest that MA representations are column-based. We hypothesize that each column in the mental abacus is mapped to a separate object representation in visual working memory, though the substructure of how each column is represented is still unknown. Consistent with this hypothesis, we found that novice abacus users showed some of the same signatures of column-based organization, suggesting that this non-linguistic format for number was adapted to the general visual capacities of its users, rather than being the result of extensive practice. Taken together, these data paint a picture of MA as a visual alternative to linguistic number representations that relies on the distinct structure of visual working memory, rather than phonological working memory (as in language-based techniques for mental arithmetic).

The example of MA goes beyond external physical representations of number like counting boards and gives strong evidence that the mental representation of exact quantity is possible in mediums other than language. Although some authors have speculated that language and exact number rely on the same computational substrate (Hauser et al. 2003), the facility and flexibility in computation shown by MA users suggests a different view. Representations of exact number can be constructed using a variety of different resources—linguistic or visual. In addition, the specific organization of the abacus/MA representation is tailored to allow computations to be decomposed into many simple operations that can be practiced independently. Although the MA addition routine requires more steps than the most common verbal algorithm, it is also more accurate because it never requires storing partial sums.

7. Conclusions. Beller and Bender (2008) write that "there may be no other domain in the field of cognitive sciences where it is so obvious that language (i.e., the verbal numeration system) affects cognition (i.e., mental arithmetic)." The data reviewed here are consistent with this contention: how a language represents large exact quantities dramatically influences how its speakers are able to store and manipulate them. For this reason, number representation presents an important case to go beyond the first order questions of the Whorfian debate—"does language influence thought"—and ask detailed questions about how language participates in constructing representations of exact number and routines for manipulating quantities. Investigations of the richness of cross-cultural variation in number systems suggest that there are major behavioral consequences that correspond to what number words a language has and how those words are structured into a count list. More generally, the form of a numerical representation (linguistic or not) structures the kinds of routines for enumeration and arithmetic that can be performed.

The data that lead to this conclusion could not have been gathered by the standard methods of cognitive psychology, nor by the standard methods of field linguistics. Many of the results cited here come from carefully controlled studies performed in the field with populations that possess culturally, linguistically, or cognitively interesting numerical representations. This generalization suggests the benefits of psycholinguistic fieldwork that combines experimental design with cross-cultural or cross-linguistic populations. Such fieldwork is especially important in the study of the diverse languages of Melanesia, since opportunities to study these languages are quickly disappearing. Future fieldwork—on number and in other domains—should take advantage of these techniques to present a fuller picture of the relations between language, culture, and cognition.

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Michael C. Frank
mcfrank@stanford.edu